

Large N_c Deconfinement Transition in the Presence of a Magnetic Field

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We investigate the effect of a homogeneous magnetic field on the thermal deconfinement transition of QCD in the large N_c limit. First we discuss how the critical temperature decreases due to the inclusion of $N_f \ll N_c$ flavors of massless quarks. Then we study the equivalent correction in the presence of an external Abelian magnetic field. To leading order in N_f/N_c , the fact that the deconfinement critical temperature decreases with the magnetic field depends solely on quarks behaving paramagnetically. Finally, we discuss the effects from a finite quark mass and its competition with magnetic effects.

Introduction. The phase diagram of strong interactions in the presence of a classical, constant, and uniform magnetic background has been attracting increasing interest in the last few years. Strong (Abelian) magnetic fields not only provide another control parameter to probe the phase structure of QCD but are also currently generated in non-central ultrarelativistic heavy ion collisions at RHIC-BNL and at the LHC-CERN. In fact, these fields are believed to be the largest ever produced since the times of the electroweak phase transition in the early universe, reaching values on the order of $B \sim 10^{19}$ Gauss ($eB \sim 6 m_\pi^2$) and even much higher [1]. Furthermore, lattice Monte Carlo simulations are not constrained by the sign problem in this case and can produce a trustworthy $T \times eB$ phase diagram, among other results. Nevertheless, the mapping of this new phase diagram is still in its infancy and presents some conflicting pictures coming from different model calculations.

In this Letter we study the behavior of the deconfining critical temperature T_c in the presence of a strong magnetic field in the large N_c limit of QCD. This provides a well-defined setup for a clean, semi-quantitative description by essentially counting powers of N_f/N_c (with N_f being the number of quark flavors) when matching pressures for the confined and deconfined sectors. In the sequel we show that, in this limit, the deconfinement temperature decreases with the magnetic field for small N_f/N_c in line with recent results from lattice QCD, provided that quarks behave paramagnetically. We also discuss how the critical temperature for the pure glue theory decreases due to the leading order correction in N_f/N_c .

All model calculations so far have suggested that sufficiently large magnetic fields, typically $eB \sim 10 m_\pi^2$, could bring remarkable modifications in the QCD phase diagram, from shifting the chiral and the deconfinement phase transition lines [2–12] to transforming the vacuum into a superconducting medium via ρ -meson condensation [13]. In particular, most model descriptions have predicted either an increase or a flat behavior for the deconfinement critical line as eB is increased to very large

values. Exceptions can be found in Ref. [2], where the critical temperature vanishes at a finite critical value of $eB_c \sim 25 m_\pi^2$, featuring the disappearance of the confined phase at large magnetic fields, and in [3], where vacuum corrections are disregarded, and T_c diminishes with eB .

The first pioneering lattice simulations [14], still with large values for the pion mass, also suggested a very mild increase of the critical temperature with eB . However, recent lattice simulations with physical masses [15] have shown that the critical temperature for deconfinement actually falls as the magnetic field increases. However, instead of falling with a rate that will bring it to zero at a given critical value of eB , it falls less and less rapidly, tending to saturate at large values of B in agreement with what one would expect from the phenomenon of magnetic catalysis [16, 17]. An exercise within the MIT bag model with the appropriate treatment of the subtleties of renormalization at finite B has shown remarkable qualitative agreement with these lattice findings with respect to the behavior of $T_c(eB)$, i.e. it decreases and saturates for very large fields [18]. To the best of our knowledge, even if known to be crude in numerical precision and missing the correct nature of the (crossover) transition, this is the only description to date that captures the correct qualitative behavior of the deconfining transition in a magnetic background.

Although a description of the deconfinement transition in the presence of an external magnetic field in terms of the MIT bag model is, of course, very simple, we believe it encodes the essential ingredient to provide a qualitative description of the behavior of $T_c \times eB$: confinement. The fact that the MIT bag model incorporates confinement (even if in its simplest fashion) seems to make it suited to describe the behavior of T_c as a function of external parameters, as hinted by a previous successful description of the behavior of the critical temperature as a function of the pion mass and isospin chemical potential, as compared to lattice data, where chiral models failed even qualitatively [19, 20]. This suggests that confinement dynamics plays a central role in guiding the

functional behavior of T_c and points towards a large N_c description of the associated magnetic thermodynamics.

Large N_c thermodynamics. The large N_c limit provides a great opportunity to study several aspects of QCD [21–24]. Feynman diagrams are reorganized according to their dependence on N_c and, when $N_c \rightarrow \infty$, only planar diagrams are relevant. The theory is still asymptotically free with a perturbative beta function defined in terms of the 't Hooft coupling $\lambda \equiv g^2 N_c$ and a renormalization group invariant energy scale Λ_{QCD} at which the associated coupling becomes strong. While confinement has not been proven in this limit, it is widely believed that in the vacuum the physical degrees of freedom are weakly interacting (since interactions go as $1/N_c$), colorless glueballs. N_f quark degrees of freedom in the fundamental representation can be added to this theory and the corresponding mesons are free when $N_c \rightarrow \infty$ while baryons become extremely heavy, $M_{baryon} \sim N_c \Lambda_{QCD}$ [23, 24].

Lattice QCD calculations [25] show that the deconfinement phase transition of pure glue $SU(N_c)$ gauge theory becomes first order when $N_c \geq 3$ [26–29] with a critical temperature $\lim_{N_c \rightarrow \infty} T_c/\sqrt{\sigma} = 0.5949(17) + 0.458(18)/N_c^2$ [30], where $\sigma \sim (440 \text{ MeV})^2$ is the string tension. The thermodynamic properties of pure glue do not seem to change appreciably when $N_c \geq 3$ [31, 32], which suggests that large N_c arguments may indeed capture the main physical mechanism behind the deconfinement phase transition of QCD.

The fact that $\lim_{N_c \rightarrow \infty} T_c/\sqrt{\sigma} \sim \mathcal{O}(N_c^0)$ and that the deconfining phase transition becomes strong first order can be readily understood using the following argument [28]. When $N_c \rightarrow \infty$ and the number of quark flavors N_f is finite such as $N_f/N_c \ll 1$, in the confined phase glueballs and mesons are very weakly interacting and, since they are colorless, they only contribute to the pressure at $\mathcal{O}(N_c^0)$. Baryons are extremely heavy in this limit and do not contribute at zero baryon chemical potential. Therefore, when $N_c \rightarrow \infty$ the only contribution to the pressure of the confined phase comes from the gluon condensate $\sim N_c^2 \Lambda_{QCD}^4$, which we write in terms of the other renormalization group invariant energy scale σ as $P_{conf} = c_0^4 N_c^2 \sigma^2$, where c_0 is a positive number of order 1. Moreover, it should be noticed that the entropy density in the confined phase vanishes.

On the other hand, asymptotic freedom implies that in the planar limit the gluon pressure is $P_{gluon}(T) = N_c^2 T^4 c_{SB}^4 f_{glue}(T/\sqrt{\sigma})$, where c_{SB} is a positive constant determined from the Stefan-Boltzmann limit and $\lim_{T/\sqrt{\sigma} \rightarrow \infty} f_{glue}(T/\sqrt{\sigma}) = 1$. The function f_{glue} depends implicitly on the 't Hooft coupling $\lambda(T)$ and, while its general form is not known when $T \sim \sqrt{\sigma}$, it should be a monotonically increasing function of T that interpolates from 0 at $T \rightarrow 0$ to 1 for $T \rightarrow \infty$. Its form can be computed using perturbation theory at sufficiently high temperatures where λ is small [33]. If $N_f = 0$, since the pressure is always continuous at any phase transition, we

see that there must be a deconfinement critical temperature defined by the condition $P_{glue}(T_c^{(0)}/\sqrt{\sigma}) = P_{conf}$ or, equivalently,

$$c_0^4 N_c^2 \sigma^2 = N_c^2 T_c^{(0)4} c_{SB}^4 f_{glue}(T_c^{(0)}/\sqrt{\sigma}), \quad (1)$$

which implies that the solution $T_c^{(0)}$ is a pure number of $\mathcal{O}(N_c^0)$ that in general cannot be computed perturbatively since it is obtained from the self-consistent equation

$$\frac{T_c^{(0)}}{\sqrt{\sigma}} f_{glue}^{1/4}\left(\frac{T_c^{(0)}}{\sqrt{\sigma}}\right) = \frac{c_0}{c_{SB}}. \quad (2)$$

Since f_{glue} increases monotonically with T one obtains that $T_c^{(0)}$ must increase with c_0 (note that the critical temperature only vanishes if $c_0 \rightarrow 0$) [34]. Lattice calculations have shown that $T_c^{(0)}/\sqrt{\sigma} \sim 0.59$ [30]. The phase transition to a Z_{N_c} symmetric deconfined phase is then of first order when $N_c \rightarrow \infty$, $N_f = 0$, and the entropy density jumps from zero to a finite number of $\mathcal{O}(N_c^2)$ at $T_c^{(0)}$.

Leading N_f/N_c corrections. The first correction to this picture appears with the inclusion of N_f flavors of massless quarks. The previous Z_{N_c} symmetry is broken explicitly in the deconfined phase because of the presence of quarks. While the $U(N_f) \otimes U(N_f) \rightarrow U(N_f)_{vector}$ pattern of (spontaneous) symmetry breaking leads to $N_f^2 - 1$ Goldstone bosons (the “pions”), their contribution to the pressure of the confined phase is of $\mathcal{O}(N_f^2 N_c^0)$, being negligible when $N_f \ll N_c$. The quark contribution to the pressure of the deconfined phase is given by $P_{quark}(T) = N_c N_f T^4 c_{qSB}^4 f_{quark}(T/\sqrt{\sigma})$, where c_{qSB} is the corresponding positive dimensionless number computed in the Stefan-Boltzmann limit and f_{quark} is a monotonically increasing function of T such that $\lim_{T/\sqrt{\sigma} \rightarrow \infty} f_{quark}(T) = 1$.

When $N_f/N_c \ll 1$ the explicit breaking of Z_{N_c} symmetry is small, slightly smoothening the phase transition into a very rapid crossover. The balance equation that defines the critical temperature $T_c^{(1)}$ modified by the quark flavors is obtained by equating the pressures $P_{conf} = P_{glue}(T_c^{(1)}) + P_{quark}(T_c^{(1)})$. Since f_{quark} is a monotonic function of T , one should expect that the critical temperature gets shifted towards smaller values. In fact, in the limit where $N_f/N_c \ll 1$ one finds the self-consistent equation

$$\frac{T_c^{(1)}}{\sqrt{\sigma}} = \frac{c_0}{c_{SB} f_{glue}^{1/4}\left(\frac{T_c^{(1)}}{\sqrt{\sigma}}\right)} \left[1 - \frac{1}{4} \frac{N_f}{N_c} \frac{c_{qSB}^4 f_{quark}\left(\frac{T_c^{(1)}}{\sqrt{\sigma}}\right)}{c_{SB}^4 f_{glue}\left(\frac{T_c^{(1)}}{\sqrt{\sigma}}\right)} \right]. \quad (3)$$

It is possible to obtain the effect of the leading order N_f/N_c correction on $T_c^{(1)}$ in terms of $T_c^{(0)}$. Keeping only

the first correction in N_f/N_c , one may take $T_c^{(1)} \mapsto T_c^{(0)}$ inside the brackets in the equation above. Since the ratio f_{quark}/f_{glue} is positive, one can define a new positive constant given by

$$c_1(N_f) \equiv c_0 \left[1 - \frac{1}{4} \frac{N_f}{N_c} \frac{c_q^4 f_{quark} \left(\frac{T_c^{(0)}}{\sqrt{\sigma}} \right)}{c_{SB}^4 f_{glue} \left(\frac{T_c^{(0)}}{\sqrt{\sigma}} \right)} \right]. \quad (4)$$

Therefore, we see that the self-consistent equation for $T_c^{(1)}$ has actually the same form as Eq. (2) and is given by

$$\frac{T_c^{(1)}}{\sqrt{\sigma}} f_{glue}^{1/4} \left(\frac{T_c^{(1)}}{\sqrt{\sigma}} \right) = \frac{c_1(N_f)}{c_{SB}}. \quad (5)$$

Thus, since $c_1(N_f) < c_0$ and f_{glue} is monotonically increasing with T , we see that the leading effect of N_f massless flavors in the large N_c limit is to decrease the critical temperature by a small amount of order N_f/N_c with respect to $T_c^{(0)}$. In other words, the addition of light quark flavors should decrease the value of the deconfinement critical temperature, which has been observed in lattice simulations [35–37].

Large N_c behavior of $T_c \times (eB)$. The same line of argument used above can be employed to show that the deconfinement critical temperature must decrease in the presence of an external magnetic field in the large N_c limit of QCD. Assuming that $N_f/N_c \ll 1$ and $m_q = 0$, the only contribution to the pressure of the confined phase that enters at $\mathcal{O}(N_c^2)$ is given by the vacuum ($B = 0$) gluon condensate $c_0^4 N_c^2 \sigma^2$. The gluon and quark condensates change in the presence of a magnetic field [38–40] but these modifications are negligible in the large N_c limit. The gluon contribution to the deconfined pressure is blind to the magnetic field, i.e., $P_{glue}(T) = N_c^2 T^4 c_{SB}^4 f_{glue}(T/\sqrt{\sigma})$.

On the other hand, the quark contribution P_{quark} feels directly the effects of the magnetic field. In fact, the regularized contribution [41] of the massless quarks to the pressure is $P_{quark}(T, eB) = N_c N_{pairs}(N_f) T^4 c_q^4 c_{SB}^4 \tilde{f}_{quark}(T/\sqrt{\sigma}, eB/T^2)$, with $N_{pairs}(N_f)/N_c \ll 1$ being the number of pairs of quark flavors with electric charges $\{(N_c - 1)/N_c, -1/N_c\}$ in units of the fundamental charge. Only the largest ($\sim N_c^0$) charge in each pair contributes to leading order in N_f/N_c .

Notice that the function \tilde{f}_{quark} is positive definite and must increase monotonically with T for a fixed value of eB until it goes to 1 in the high temperature limit $T \gg \sqrt{\sigma}, eB$. Given our previous analysis for the case where $N_f \neq 0$ and $B = 0$, one should expect that the critical temperature as a function of the magnetic field, $T_c(eB)$, must decrease with respect to $T_c^{(0)}$ by an amount of $\mathcal{O}(N_f/N_c)$.

This can be seen directly by equating the pressures at T_c

$$c_0^4 N_c^2 \sigma^2 = N_c^2 T_c^4 c_{SB}^4 f_{glue} \left(\frac{T_c}{\sqrt{\sigma}} \right) + N_c N_{pairs}(N_f) T_c^4 c_q^4 c_{SB}^4 \tilde{f}_{quark} \left(\frac{T_c}{\sqrt{\sigma}}, \frac{eB}{T_c^2} \right) \quad (6)$$

and noticing that, since the left-hand side of the equation above is fixed (and independent of B at this order), the addition of the quark contribution on the right-hand side must lead to a decrease of the critical temperature by an amount of order N_f/N_c . In fact, the solution of the equation above for $T_c(eB)$, to leading order in N_f/N_c , is

$$\frac{T_c(eB)}{\sqrt{\sigma}} f_{glue}^{1/4} \left(\frac{T_c(eB)}{\sqrt{\sigma}} \right) = \frac{c_2(N_{pairs}, eB)}{c_{SB}}, \quad (7)$$

where we defined

$$c_2(N_{pairs}, eB) \equiv c_0 \times \left[1 - \frac{1}{4} \frac{N_{pairs}(N_f)}{N_c} \frac{c_q^4 c_{SB}^4 \tilde{f}_{quark} \left(\frac{T_c^{(0)}}{\sqrt{\sigma}}, \frac{eB}{T_c^{(0)2}} \right)}{c_{SB}^4 f_{glue} \left(\frac{T_c^{(0)}}{\sqrt{\sigma}} \right)} \right] \quad (8)$$

Since $c_2(N_{pairs}, eB) < c_0$, the same arguments used before guarantee that $T_c(eB)/T_c^{(0)} < 1$ by an amount $\sim N_f/N_c$. Therefore, one concludes that, in the presence of an external magnetic field, the deconfinement critical temperature decreases with respect to its value for pure glue in the large N_c limit of QCD. Whether $T_c(eB)$ is also lower than the critical temperature in the presence of N_f/N_c flavors of massless quarks at $B = 0$, $T_c^{(1)}$, requires that $c_2(N_{pairs}, eB) < c_1$. This implies that quarks should behave paramagnetically, with positive magnetization $M(T_c, eB) = dP_{quark}(T_c, eB)/dB \geq 0$ for all values of B .

For a free gas implementation of the deconfined phase $f_{glue} = 1$ and, in the limit of strong magnetic fields $eB/T^2 \gg 1$, one finds that $\tilde{f}_{quark} \sim eB/T_c^2$ [18]. Thus, in this case the magnetic suppression of the deconfinement critical temperature goes like $eB N_{pairs}/(N_c \sigma)$. In fact, this simple implementation in the limits of low and high magnetic fields provides a scenario in which the slope in $T_c(eB)$ decreases for large fields, as illustrated in Fig. 1.

An eventual saturation of T_c as a function of eB , as observed on the lattice [15] and in model calculations [18], cannot be obtained using the limits discussed in this paper in a general fashion. The specific form of T_c as a function of eB depends on the non-perturbative functions f_{glue} and \tilde{f}_{quark} . In fact, in the large N_c limit, our results indicate that $T_c(eB)$ can only be a flat curve if \tilde{f}_{quark} is such that $M(T_c, eB)$ is positive but it vanishes for large fields. In this scenario, the most likely explanation for the nearly flat curve found in the $N_c = N_f = 3$ lattice study performed in Ref. [15] is a net cancellation effect

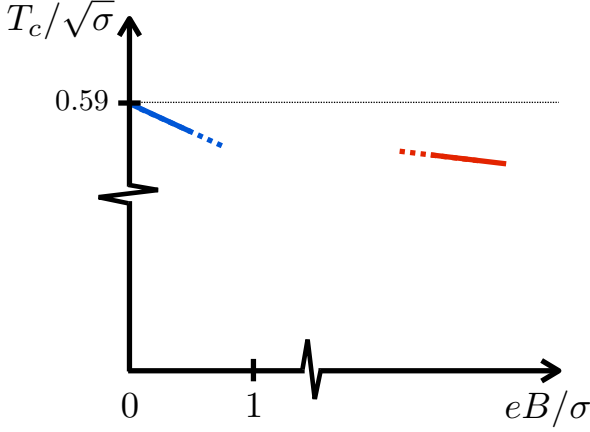


FIG. 1: Cartoon of the $T_c \times eB$ phase diagram in the large N_c limit, using the approximation of free deconfined quarks and gluons. The numerical value 0.59 shown in the plot was extracted from Ref. [30].

that occurs for sufficiently large fields due to a magnetic field dependent contribution to the pressure below the phase transition.

Quark mass effects. When $m_q \neq 0$ the pressure of the confined phase is increased by the quark contribution to the vacuum trace anomaly, $N_c N_f m_q (-\langle \bar{q}q \rangle)$, where we used the fact that the quark condensate is negative. This is equivalent to a small positive shift of c_0 , in leading order in N_f/N_c :

$$c_0 \xrightarrow{m_q \neq 0} c_{m_q} = c_0 \left(1 + \frac{1}{4} \frac{N_f}{N_c} \frac{m_q}{\sqrt{\sigma}} \frac{(-\langle \bar{q}q \rangle)}{c_0^4 \sigma^{3/2}} \right) \quad (9)$$

In the deconfined phase only the quark pressure will be affected by the quark mass effects, decreasing e.g. in perturbation theory [33]. In a large temperature expansion, we may write: $f_{quark} \mapsto f_{quark} - c_3 m_q^2/T^2$, where c_3 is positive. Therefore, the critical temperature computation in this massive case follows the same steps that led to Eqs. (4) and (5), with the substitutions $c_0 \mapsto c_{m_q} > c_0$ and $f_{quark}|_{m_q=0} \mapsto f_{quark}|_{m_q=0} - c_3 m_q^2/T^2 < f_{quark}|_{m_q=0}$. As a consequence, $c_1(N_f, m_q) > c_1(N_f, m_q = 0)$ and $T_c^{(m_q)}$ is higher than its massless counterpart, $T_c^{(1)}$. Interestingly enough, however, the corrections to c_0 and f_{quark} are respectively $\sim m_q/\sqrt{\sigma}$ and $\sim (m_q/T_c^{(0)})^2$, being extremely small for reasonable values of quark masses, $m_q \ll \sqrt{\sigma}, T_c^{(0)}$. Therefore, in this large N_c regime, we find that the critical temperature as a function of m_q is essentially flat, in line with lattice results [42, 43].

Of course, the explicit dependence of T_c with respect to the quark mass (or equivalently the pion mass) will also depend on the details of the functions f_{glue} and f_{quark}

as well as the quark condensate. In the study performed in [19, 20] within an effective model implementation of the $N_c = 3$ and $N_f = 2$ deconfined phase, $T_c/\sqrt{\sigma}$ was found to be nearly constant with respect to variations in the pion mass.

In the presence of a magnetic field, the quark condensate and its influence on T_c are unaltered at this order in N_f/N_c , while the quark pressure receives magnetic contributions, becoming $\hat{f}_{quark}(T/\sqrt{\sigma}, m_q/T, eB/T^2)$. Therefore, the critical temperature $T_c^{(2, m_q)}$ is the solution of Eq. (7) with c_2 replaced by

$$\frac{c_2(N_{pairs}, eB, m_q)}{c_{m_q}} = \left[1 - \frac{1}{4} \frac{N_{pairs}(N_f)}{N_c} \times \frac{c_{qSB}^4 \hat{f}_{quark} \left(\frac{T_c^{(0)}}{\sqrt{\sigma}}, \frac{m_q}{T_c^{(0)}}, \frac{eB}{T_c^{(0)2}} \right)}{c_{SB}^4 f_{glue} \left(\frac{T_c^{(0)}}{\sqrt{\sigma}} \right)} \right]. \quad (10)$$

While $c_{m_q} > c_0$, which enhances the critical temperature, the brackets behave according to the quark pressure in the presence of B . Again, if quarks behave paramagnetically, \hat{f}_{quark} increases and there will be a competition between mass and magnetic effects, which may lead to a non-monotonic behavior of the critical temperature as the quark mass is varied.

Final comments. It would be interesting to extend the discussion about the magnetic effects on the deconfinement critical temperature to the Veneziano limit of QCD. In this case, one could also study whether chiral symmetry restoration coincides with the deconfinement transition when $N_f, N_c \rightarrow \infty$ in the presence of an external magnetic field.

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